On the Existence and the number of Pairwise stable networks

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Résumé. A weighted network g is defined by N nodes (the agents), and for each non ordered pair of nodes denoted ij, some weighted link denoted g_{ij}(represented by a number in $[0,1]$). It can represent the intensity of the relationship between i and j. Each agent i has some preference on the set $\mathcal{G}$ of weighted networks, represented by a payoff function $v_i : \mathcal{G} \to \mathbf{R}$. A weighted network $g \in \mathcal{G}$ is *pairwise stable* (see [1]) if:

(i) for every $ij \in \mathcal{L}$, for every $x \in [0, g_{ij}[$, $v_i((x, g_{-ij})) \leq v_i(g)$.
(ii) for every $ij \in \mathcal{L}$, for every $x \in]g_{ij}, 1]$, if $v_i((x, g_{-ij})) > v_i(g)$ then $v_j((x, g_{-ij})) < v_j(g)$ (resp. $v_j((x, g_{-ij})) \leq v_j(g)$).

(in this definition, (x, g_{-ij}) denote the weighted network g where weight g_{ij} has been replaced by x).
The first condition says that in g, no agent has some interest to unilateraly decrease the weight of some of his link, and the second condition says that no two agents have some interest to bilateraly increase the intensity of their relationship.

The concept has been intensively used in Network theory since 20 years to represent a stable network in any *formation network process*. But there is no general existence result in the spirit of Nash existence result. We give such result and we also raise the following questions (and solve some of them):

- Can pairwise stable network be obtained as a Nash equilibrium of some "natural" game ?
- Is there some notion of *mixed-strategy* pairwise stable network, similar to game theory ?
- What can be said, in general, about the number of pairwise stable networks ?
- What are the natural dynamics related to pairwise stable networks ?

Mots-clefs : Pairwise stable networks, existene, generic number.

Références

[1] Philippe Bich, Lisa Morhaim. *On the existence of pairwise stable weighted networks.* Submitted.

[2] Matthew Jackson, Asher Wolinsky. *A Strategic Model of Social and Economic Networks.* Journal of Economic Treory, 1996.