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Semistability Theory of Nonsmooth Dynamical Systems

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Résumé. We are interested in the systems of the form

$$\dot{x}(t) + F(x(t)) \in -\partial\varphi(x(t)),$$

Where $F : \varphi : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function and $\varphi : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ is a proper, convex and lower semi-continuous function. The aim of this talk is to consider an alternative notion of stability called semistability which, in certain sense, lies between stability and asymptotic stability. More precisely, an equilibrium is semistable if it is Lyapunov stable, and every trajectory starting in a neighborhood of the equilibrium converges to a (possibly different) Lyapunov stable equilibrium. It can be seen that, for an equilibrium, asymptotic stability implies semistability, while semistability implies Lyapunov stability. In addition to semistability, it is desirable that a dynamical system that exhibits semistability also possesses the property that trajectories that converge to a Lyapunov stable system state must do so in finite time rather than merely asymptotically. This is the so called finite-time semistability.
Two approaches will be used to treat these two notions of convergence and semistability. In the first one, the results are Lyapunov-based and are obtained without any assumptions of sign definiteness on the Lyapunov function. In the second one, we use a condition based on nontangency between the vector field and invariant or negatively invariant subsets of the level or sublevel sets of the Lyapunov function or its derivative.

Mots-clefs : Nonsmooth Dynamics, Semistability theory, Finite-time semistability

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