Journées SMAI MODE 2018

RESTORING SHORT-PERIOD OSCILLATIONS OF THE MOTION OF AVERAGED OPTIMAL CONTROL SYSTEMS

Lamberto Dell'Elce
Inria, France

Jean-Baptiste Caillau
Université de la Côte d'Azur, France

Jean-Baptiste Pomet
Inria, France

Résumé. Averaging is a valuable technique to gain understanding in the long-term evolution of dynamical systems characterized by slow and fast dynamics. Short period variations of averaged trajectories can be restored *a posteriori* by means of a near-identity transformation that is a function of both the averaged slow and fast variables. Recent contributions in optimal control theory prove that averaging can be applied to the dynamical system resulting from the necessary conditions for optimality. The present talk extends these results by discussing the evaluation of short-period variations of the adjoint variables. First, the classical approach is shown to be inadequate when applied to the assessment of the adjoints of slow variables because of the peculiar form of their equations of motion. Hence, a consistent transformation is developed, such that variations of the adjoints of fast and slow variables are evaluated in sequence. A simplified transformation is finally obtained when a single fast variable is considered. The methodology is applied to a time-optimal low-thrust orbital transfer.

Mots-clefs : Averaging, Pontryagin's principle, osculating elements, symplectic transformation.

When the state of a dynamical system can be decomposed into fast-periodic and slow components, averaging the equations of motion over the instantaneous period of the fast variables is a valuable practice to simplify the dynamics and gain understanding in the long-term evolution of the flow [1]. If the estimation of the short-period variations of the trajectory is envisaged, a near-identity transformation that restores them *a posteriori*, i.e., as a function of the state of the averaged system, can be developed. In the general multidimensional case, this transformation consists of a linear combination of the multivariate Fourier coefficients of slow states' dynamical equations.

Recent contributions in optimal control show that the averaging technique can be applied to the extremal flow of minimum energy [2] and time [3] problems. These results are obtained by proving that the adjoints of slow variables, named p_s hereafter, have slow dynamics, too. In addition, the magnitude of the adjoints of fast variables, named p_f hereafter, is demonstrated to be small during the entire trajectory.

Averaging the controlled system facilitates the challenging task of providing a reliable initial guess to shooting algorithms. The quality of this guess can be theoretically enhanced by applying the aforementioned transformation at the beginning of the maneuver. In this way, although the trajectories of the averaged and original systems are emanated from different points of the phase space, the early motion of the original system fluctuates about its averaged counterpart[1], so that the control vector is more accurately predicted and, consequently, the drift between the long-term evolution of the two systems is reduced and the estimation of the fast dynamics is greatly improved. Unluckily, the reconstruction of p_s is unsatisfactory if the classical algorithm is exploited.

The first part of the talk is aimed at providing insight into this result by showing that short-period variations of p_f (although small) are responsible of a non-negligible contribution to the dynamics of p_s and, as such, neglecting them when evaluating the Fourier coefficients is the source of the error.

Hence, a new two-step transformation is proposed. First, the variations of p_f as a function of the fast variables are reconstructed by means of the classical algorithm. When the system has a single fast variable, p_f can be straightforwardly evaluated by using an explicit equation that can be interpreted as a first-order matching between the Hamiltonian of the averaged and original systems. Eventually, the fast oscillations of p_s and of the slow states are estimated by accommodating the information provided by the reconstructed p_f.

The developments are illustrated by means of a time-optimal low-thrust orbital transfer in the planar circular restricted Earth-Moon system. Here, slow variables are the equinoctial elements [4] defining the shape and orientation of the orbit, namely the semilatus rectum and eccentricity vector, whereas the mean longitude of the satellite and of the Moon are the two fast variables. Figure 1 depicts the evolution of the adjoint of the semilatus rectum during the first orbital period of the Moon. Dashed and dash-dotted lines are the trajectories of the averaged and original systems emanated from the same initial state, respectively. A monotonic difference exists between these two curves. When the classical transformation is used, the reconstructed p_s (dotted curve) does not accurately reproduce the motion emanated from the transformed initial conditions (solid curve). On the contrary, a remarkable improvement is obtained when the proposed methodology is exploited, as revealed by the enhanced matching between the solid and dotted curves.

Références

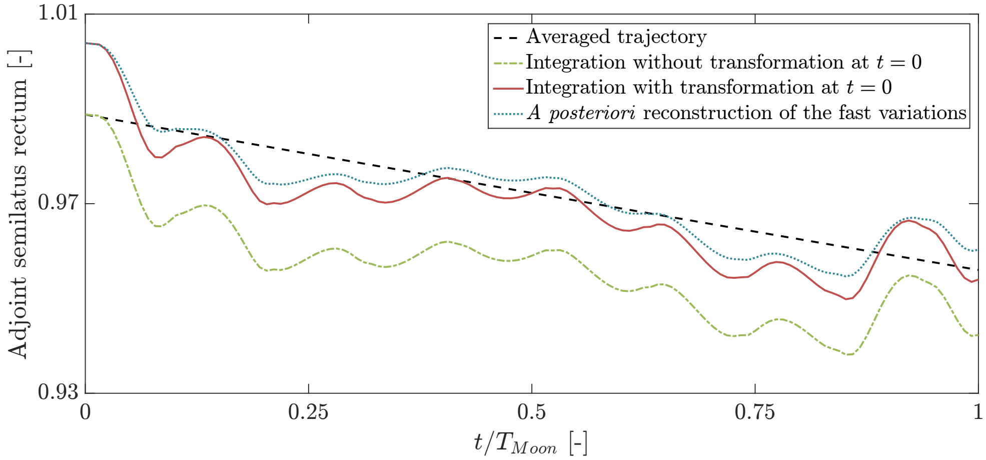
[1] P. Lochak, C. Meunier, *Multiphase Averaging for Classical Systems*, Springer, 1988.

[2] B. Bonnard, J. B. Caillau, *Forum Mathematicum. Geodesic flow of the averaged controlled Kepler equation*, Walter de Gruyter GmbH, 2009.

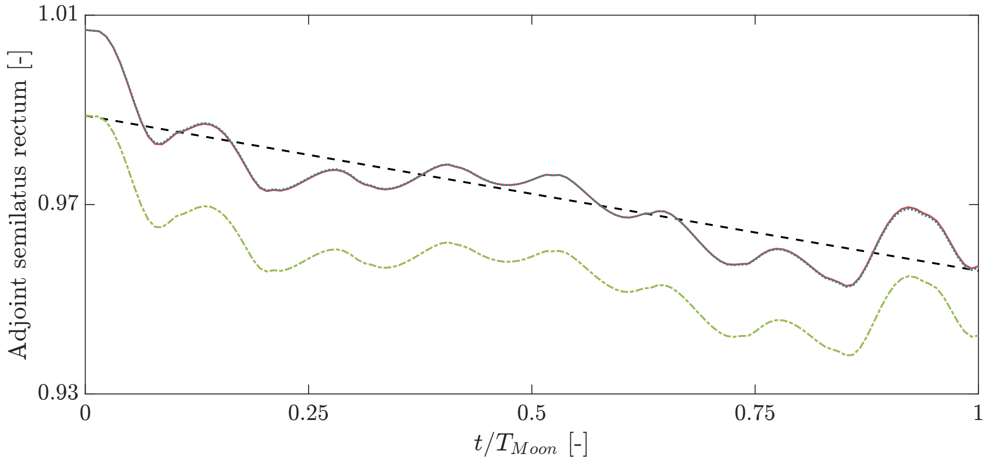
[3] A. Bombrun, J. B. Pomet, *SIAM Journal on Control and Optimization. The averaged control system of fast-oscillating control systems*, SIAM, 2013.

[4] R. H. Battin, *An Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition*, American Institute of Aeronautics and Astronautics (AIAA), 1999.

[1] This statement is clarified by comparing the trajectories emanated from unchanged and transformed initial conditions in Figure 1 (solid and dash-dotted lines, respectively).



(a) Classical transformation



(b) Proposed transformation

Figure 1: Trajectory of the normalized adjoint of the orbital semilatus rectum. Table 1 lists the numerical values used to generate this trajectory.

Table 1: Numerical values used to generate Figure 1.

Constants		
Earth's gravitational parameter		$3.986 \cdot 10^5$ km^3 s^{-2}
Moon's gravitational parameter		$4.905 \cdot 10^3$ km^3 s^{-2}
Moon's orbital radius, r_{Moon}		$384.4 \cdot 10^3$ km
Thrust to mass ratio		10^{-2} N / 1500 kg
Initial Averaged conditions	**State**	**Adjoint**
Orbital semilatus rectum to r_{Moon} ratio	0.1122	0.989
Eccentricity vector, x component	0.7	0.148
Eccentricity vector, y component	0	0
Mean longitude	344 deg	0
Moon's mean longitude	180 deg	0