Journées SMAI MODE 2018

Games with information. Witsenhausen intrinsic model

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Résumé. We study the concept of information. Regarding a stochastic control problems model by Witsenhausen we look at a usual game theoretical problem without knowing a priori the order in which decisions are made by agents. This approach yields configurations that are not covered by classical theory. We are as well studying possible systems and classify them using four types of orderings. We show that the extensive form model is embedded into Witsenhausen model with the potential of embedding Bayesian games as well.

Mots-clefs : Control; Extensive forms; Sequential decision theory; Trees; Information.

In a context of competition, information – who knows what and before whom – plays a crucial role. There was proposed a model by Witsenhausen [4] to handle, in all generality, information in stochastic control problems. It allows to look at a problem without knowing a priori the order in which decisions are made by agents. This approach yields configurations that are not covered by classical theory. An exogenous information structure determines the order in which agents will take their decisions – depending on some certain conditions [2] there can be one possible ordering or several or none. To classify the systems Witsenhausen introduces equivalence relations [6]. To extend Witsenhausen intrinsic model (WIM) to game theory, we need to translate the standard notions of the field in this framework. Games defined in this framework will be designed as Witsenhausen games or, equivalently, intrinsic games. As pointed out by Witsenhausen, the difficulties in specifying the information structure of a game were faced and overcome in the early days of Game Theory through the introduction of extensive form games. The extensive form is the most richly structured way to describe game situations. The definition of the extensive form that is now standard in most of the literature on Game Theory, is due to Kuhn [3], who modified the earlier definition used by von Neumann and Morgenstern. We show that extensive form is embedded into WIM with the potential of embedding Bayesian games as well. In a nutshell, an intrinsic form game is defined by:

- a set of **agents** partitioned by a **player** equivalence relation
- a **random set** Ω and for each agent, an **action set** $\mathbb{U}_a$, all equipped with σ**-fields**
- for each agent an **information field**
- for each agent, a set of measurable and **admissible policies**, which are mapping from the history to the decision set
- the solvability/measurability property needs to be satisfied, and all for the introduction of a **measurable solution map**

- for each player, a measurable and bounded **criterion** and a **prior probability distribution**

On the other hand, the tree structure that we as well propose to revisit may be generalized allowing for having infinite actions available at each locus of taking a decision. At the same time, it is possible to lift the limitation on the finitness of the tree's length. It requires to apply some fresh set-theoretical approaches rewrtitting a tree as a set of subsets of paths (plays) from the root to a leaf bequeathing the tree structure. In these approaches we rely on the existing literature [1]. It may be also fruitful looking in the direction of the most general tree to be embedded in the WIM.
The main contribution is in revisiting the basic concepts of game theory – Bayesian game, game in extensive form, backward induction, subgame – to grasp them in the WIM framework. We outline how the WIM offers an intrinsic way to describe and classify classic specimen of principal-agent models – like Stackelberg leadership model, moral hazard, adverse selection, signaling. We discuss a possible research program.

Références

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